Exercise 03 Tasks. 24.04.2024

Task 1. Decomposition of the Velocity field.

$\underline{v}$: Velocity Field

$\underline{\underline{L}}$: $\nabla \underline{v}$: Velocity Gradient

$\underline{\underline{D}}$: Rate of Deformation Tensor $= \frac{1}{2}(\nabla \underline{v} + \nabla \underline{v}^T)$

$\underline{\underline{W}}$: Spin Tensor $= \frac{1}{2}(\nabla \underline{v} - \nabla \underline{v}^T)$

$$\underline{\underline{D}} = \text{sph}(\underline{\underline{D}}) + \text{dev}(\underline{\underline{D}})$$

$$\text{sph}(\underline{\underline{D}}) = \frac{1}{3}\text{tr}(\underline{\underline{D}})\underline{\underline{I}} \qquad \text{dev}(\underline{\underline{D}}) = \underline{\underline{D}} - \text{sph}(\underline{\underline{D}})$$

a) $\underline{v} = \begin{pmatrix} a \\ 0 \end{pmatrix} \underline{e}_i$

$$\nabla \underline{v} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j$$

$$\underline{\underline{D}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j \qquad \underline{\underline{W}} = \underline{\underline{0}}$$

$$\text{sph}(\underline{\underline{D}}) = \underline{\underline{0}} \qquad \text{dev}(\underline{\underline{D}}) = \underline{\underline{0}}$$

---

b) $\underline{v} = \begin{pmatrix} 2y+10 \\ 0 \end{pmatrix}$

$$\nabla \underline{v} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j \qquad \underline{\underline{D}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j \qquad \underline{\underline{W}} = \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j$$

$$\text{sph}(\underline{\underline{D}}) = \underline{\underline{0}} \qquad \text{dev}(\underline{\underline{D}}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underline{e}_i \otimes \underline{e}_j$$

c) $\underline{v} = \begin{pmatrix} bx \\ -by \end{pmatrix}$

$$\nabla \underline{v} = \begin{bmatrix} b & 0 \\ 0 & -b \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\underline{\underline{D}} = \begin{bmatrix} b & 0 \\ 0 & -b \end{bmatrix} \underline{e_i} \otimes \underline{e_j} \qquad \underline{\underline{W}} = \underline{\underline{0}}$$

$$\mathrm{sph}(\underline{\underline{D}}) = \underline{\underline{0}} \qquad \mathrm{dev}(\underline{\underline{D}}) = \begin{bmatrix} b & 0 \\ 0 & -b \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

d) $\underline{v} = \begin{pmatrix} cx \\ cy \end{pmatrix}$

$$\nabla \underline{v} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\underline{\underline{D}} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \underline{e_i} \otimes \underline{e_j} \qquad \underline{\underline{W}} = \underline{\underline{0}}$$

$$\mathrm{sph}(D) = \frac{1}{2} \mathrm{tr}(\underline{\underline{D}}) \underline{\underline{I}} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \underline{e_i} \otimes \underline{e_j} \qquad \mathrm{dev}(\underline{\underline{D}}) = \underline{\underline{0}}$$

e) $\underline{v} = \begin{pmatrix} dy \\ dx \end{pmatrix}$

$$\nabla \underline{v} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\underline{\underline{D}} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j} \qquad \underline{\underline{W}} = \underline{\underline{0}}$$

$$\mathrm{sph}(\underline{\underline{D}}) = \underline{\underline{0}} \qquad \mathrm{dev}(\underline{\underline{D}}) = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

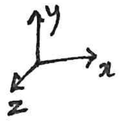
f) $v = \begin{pmatrix} -fy \\ fx \end{pmatrix}$

$$\nabla \underline{v} = \begin{bmatrix} 0 & -f \\ f & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\underline{\underline{D}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j} \qquad \underline{\underline{W}} = \begin{bmatrix} 0 & -f \\ f & 0 \end{bmatrix} \underline{e_i} \otimes \underline{e_j}$$

$$\mathrm{sph}(\underline{\underline{D}}) = 0 \qquad \mathrm{dev}(\underline{\underline{D}}) = \underline{\underline{0}}$$

Task 2 **Vorticity.** Calculate the vorticity for the given velocity fields



$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Vorticity: $\nabla \times \underline{v}$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$i\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + j\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + k\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

a) $v = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \Rightarrow \nabla \times \underline{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \underline{e_i}$ b) DIY

c) $v = \begin{pmatrix} bx \\ -by \end{pmatrix} \Rightarrow \nabla \times v = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \underline{e_i}$ d), e) DIY

0

f) $v = \begin{pmatrix} -fy \\ fx \end{pmatrix} \Rightarrow \nabla \times v = \begin{pmatrix} 0 \\ 0 \\ 2f \end{pmatrix} \underline{e_i}$

0